

ME 243

Mechanics of Solids

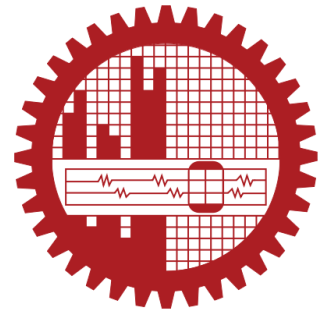
Lecture 2: Stress and Strain

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Tension test of a material

- In a tension test, a specimen is gripped between the jaws of a testing machine.
- Values of load and the elongation in a specified length, called *gauge length* are observed simultaneously.
- Stresses are calculated by dividing the forces by the cross sectional area of the specimen and strains are calculated by dividing the elongation by the gauge length.
- Stress-strain curve is plotted to find the mechanical properties of the specimen material.

Tension test of a material

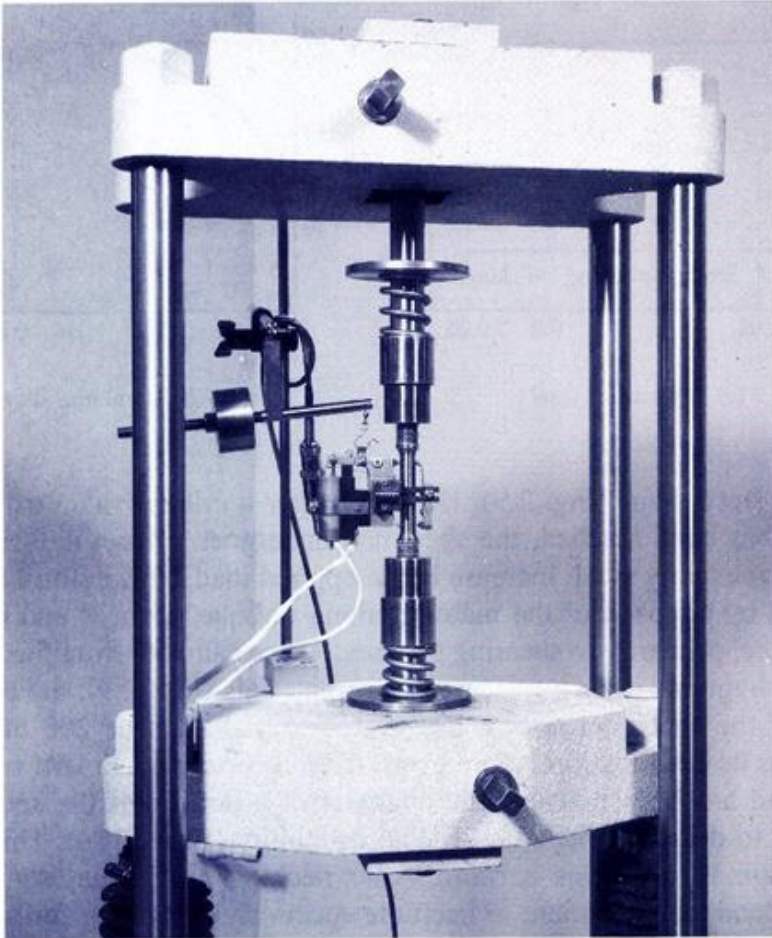


Fig. This machine is used to test tensile test specimens,

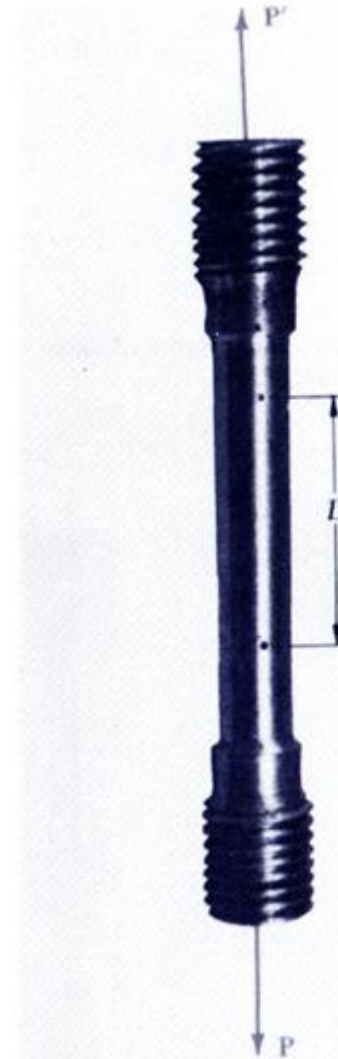


Fig. Test specimen with tensile load.

Tension test of mild steel



Tensile test specimen(mild steel) after breaking

Some definitions

- **Ductility:** It is a property that enables a material to undergo plastic deformation due to tensile stress.

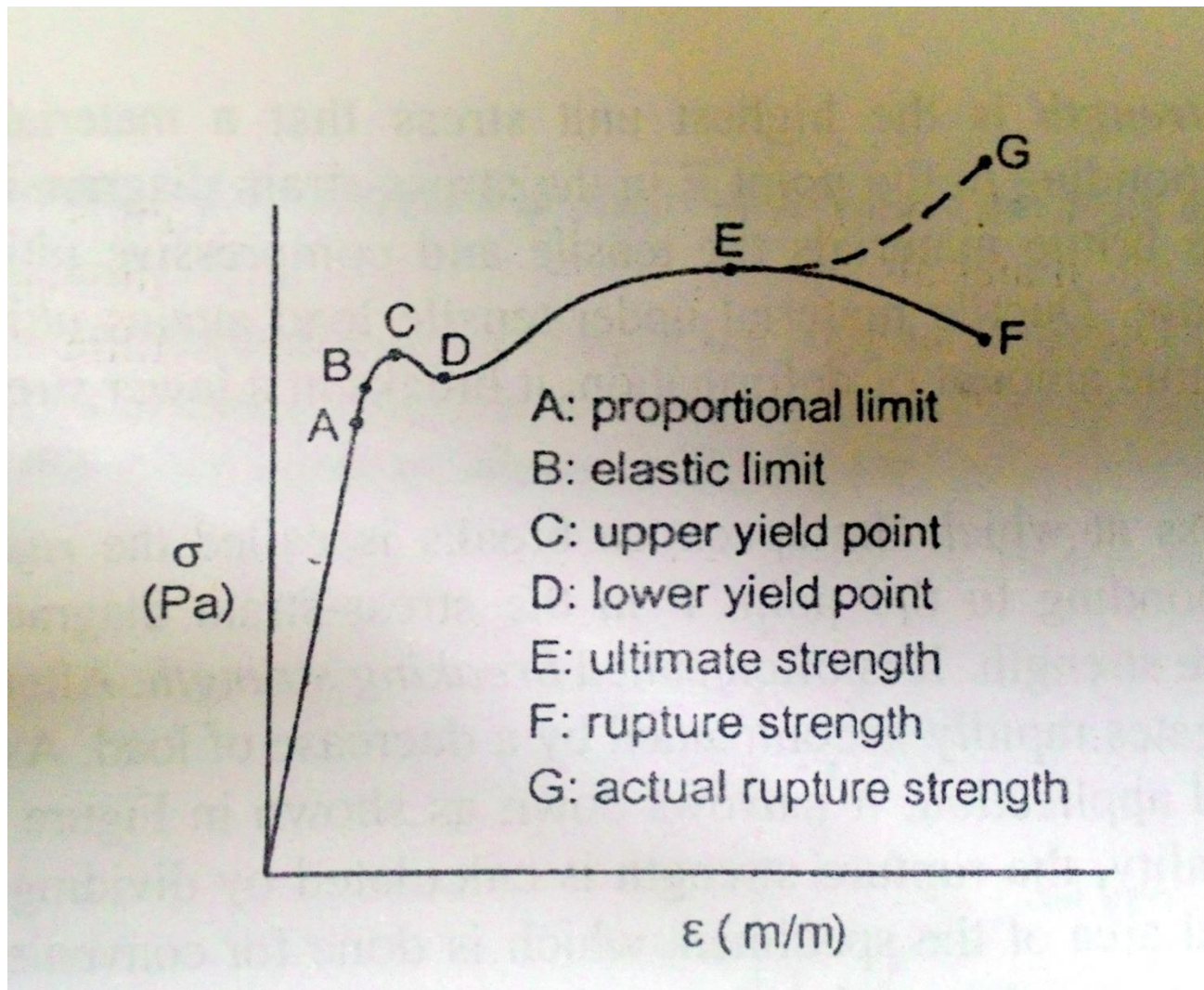
Materials that show ductility are called ductile materials (e.g. mild steel, aluminum, gold etc.)

Ductility can be expressed by

1. Percentage elongation
2. Percentage reduction in area.

- **Brittleness:** It is the tendency of a material to fracture or fail without considerable deformation.
e.g. cast iron, concrete, ceramics etc.

Stress-strain diagram of mild steel



Strain-strain diagram of mild steel

- **Proportional limit:** The limit upto which stress is proportional to strain (point A).
- **Elastic limit:** The limiting stress beyond which a material cannot be stressed without causing a permanent deformation (point B).
- **Yield point:** It is the point at which there is an appreciable elongation or yielding of the material without any corresponding increase of load.

The stress corresponding to point C is the **upper yield point**.

The stress corresponding to point D is the **lower yield point**.

The stress corresponding to yield point is called **yield strength**. Lower yield point is used to determine the yield strength of a material.

Strain-strain diagram of mild steel

- **Ultimate strength:** It is the highest stress that a material can withstand (point E).
- **Rupture or Breaking strength:** the stress at which the specimen breaks.
- **Actual rupture or breaking strength:** If the load is divided by the true section area as the specimen narrows down after the ultimate strength, the stress will follow the dotted line as shown in figure.

If the rupture area is measured after the failure and the load is divided by this area, the stress obtained is called **actual rupture of breaking strength**.

Hooke's law

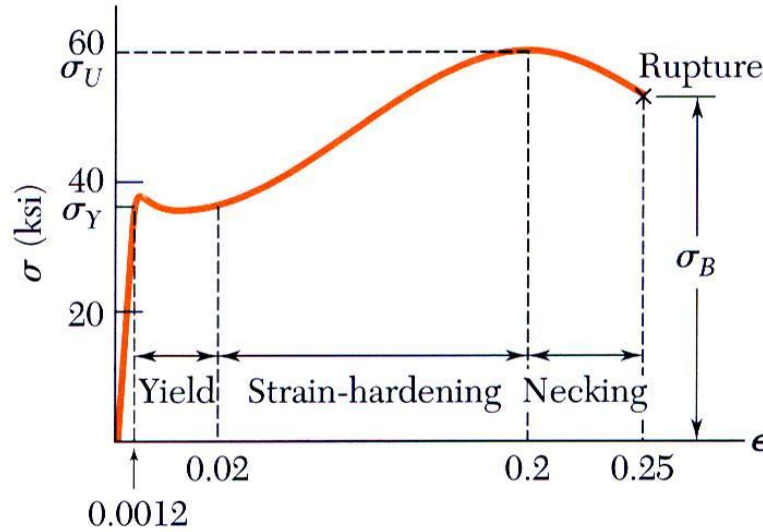
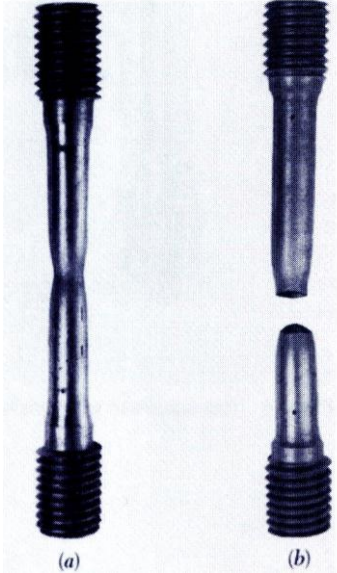
- Slope of the stress-strain curve within the proportional limit is called **modulus of elasticity** or **Young's modulus**.

$$\text{Modulus of elasticity, } E = \frac{\sigma}{\varepsilon}$$

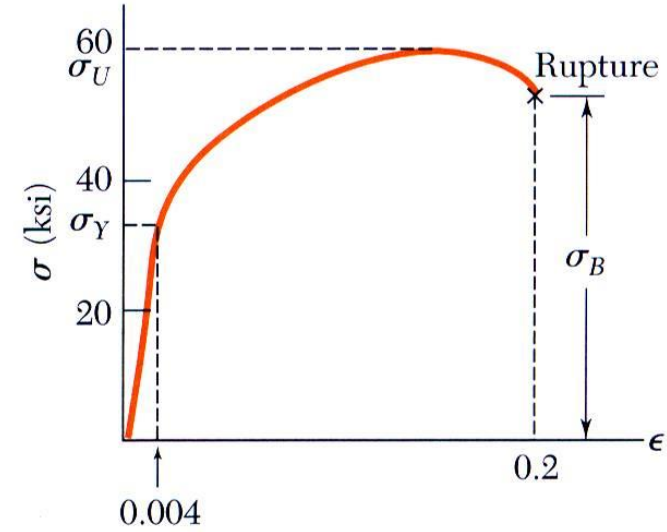
or, $\sigma = E\varepsilon$

- So, within the proportional limit, the stress is proportional to strain. It is known as **Hooke's law**.

Stress-strain diagram for other materials



(a) Low-carbon steel



(b) Aluminum alloy

Stress-strain diagram for other materials

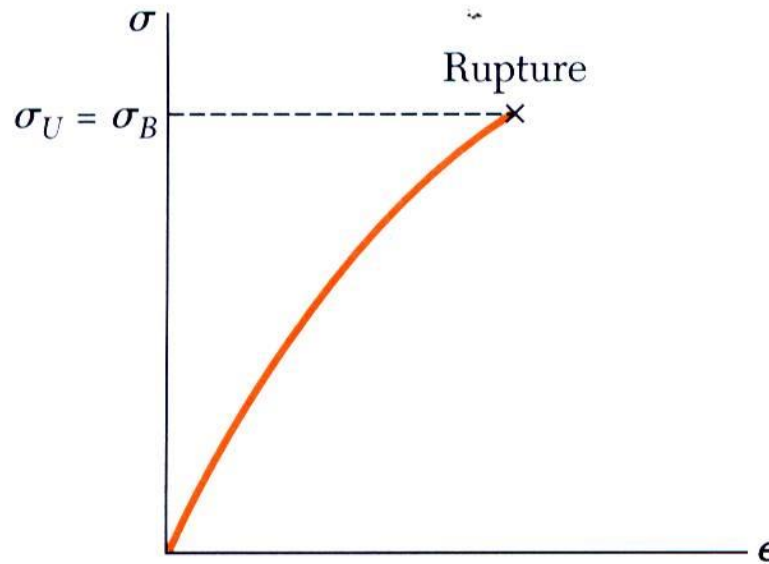
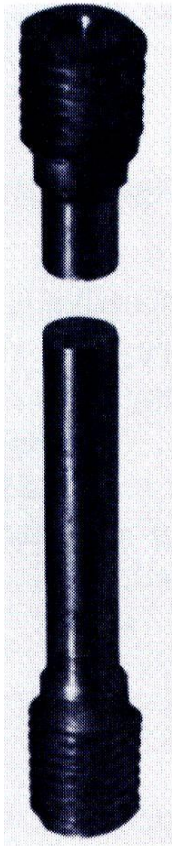


Fig. Stress-strain diagram for a typical brittle material.

Brittle material fails at ultimate stress. No yield point is found.

Stress-strain diagram for other materials

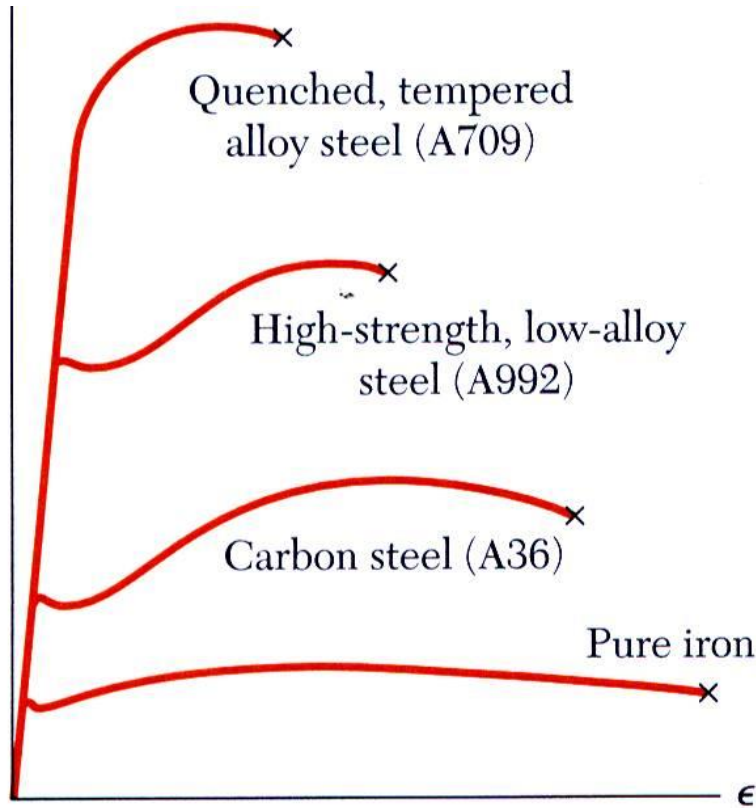
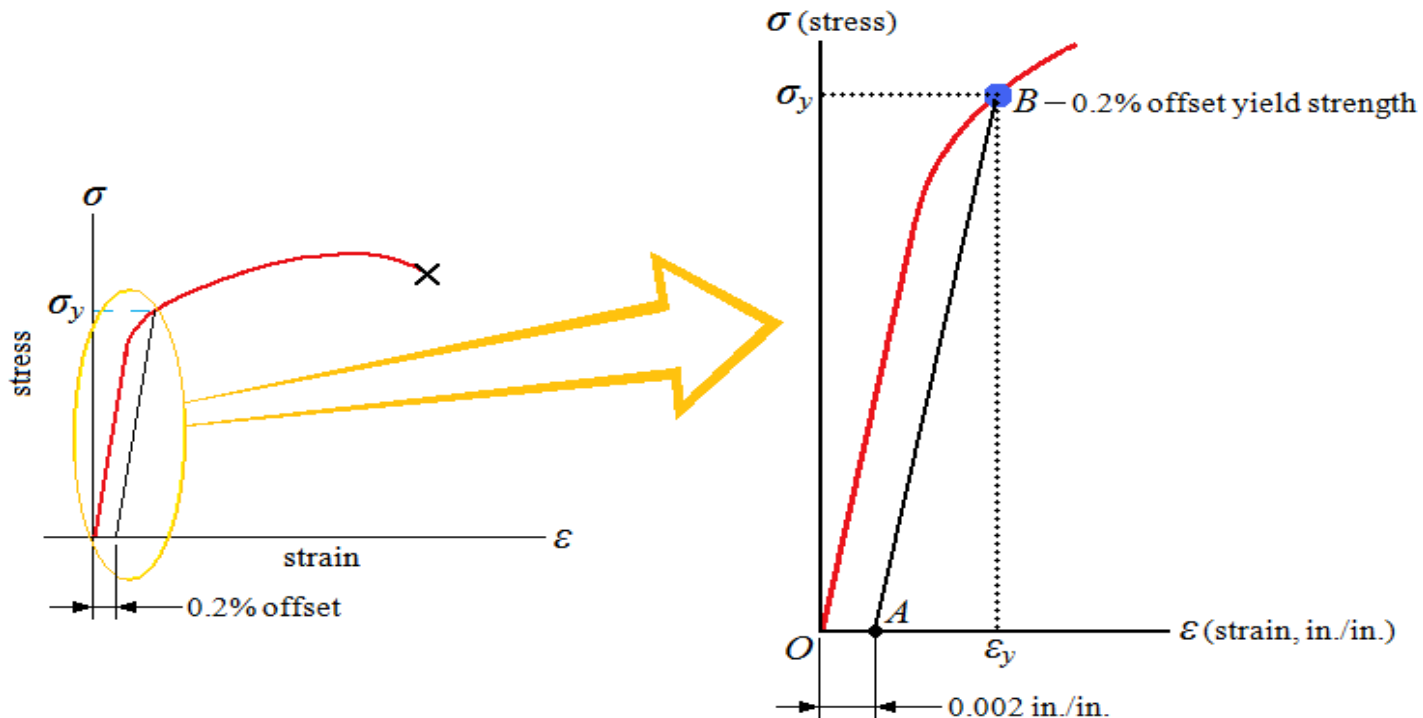


Fig. Stress-strain diagrams for iron and different grades of steel.

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Offset method for yield strength

For some ductile materials whose yield point are not well defined, the yield point for such materials is determined by offset method. This is done by drawing a parallel line to the initial tangent of the stress-strain diagram. The parallel line is drawn at an arbitrary offset strain, usually 0.2% (0.002 mm/mm or in/in). The yield point corresponds to the intersecting point of the parallel line drawn with the stress-strain curve.



Deformations Under Axial Loading

- From Hooke's Law:

$$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E} = \frac{F}{AE}$$

- From the definition of strain:

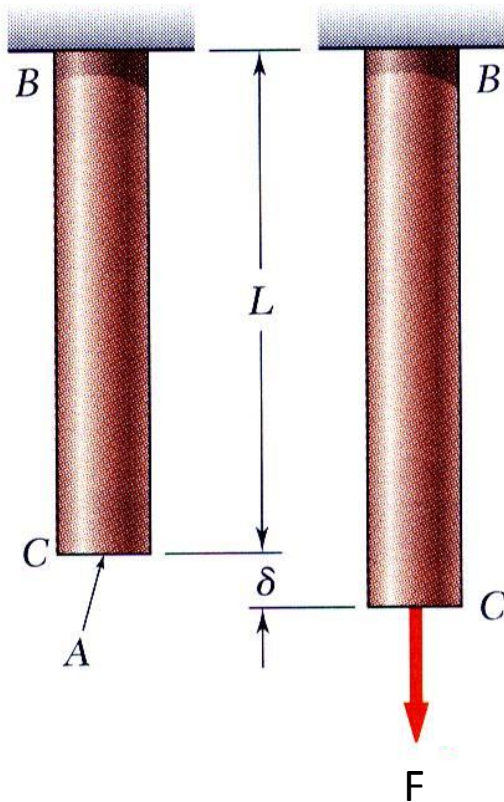
$$\varepsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

$$\delta = \frac{FL}{AE}$$

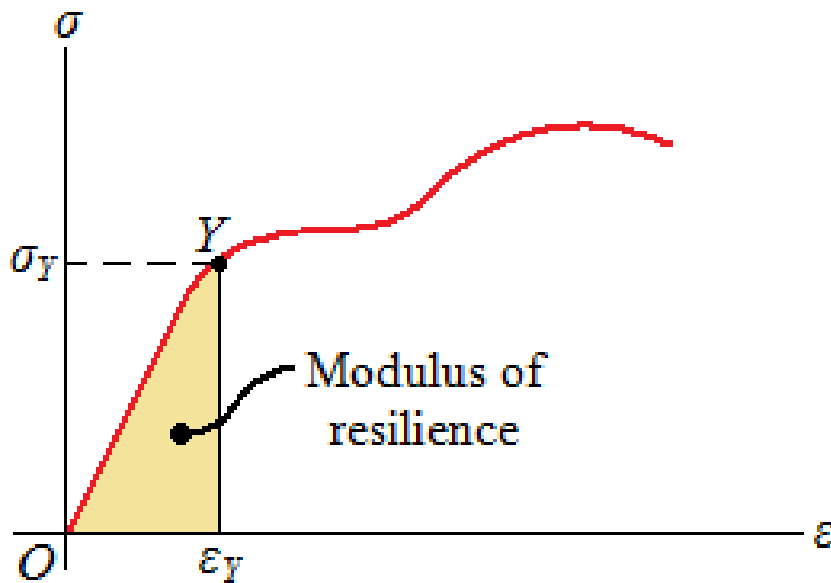
- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{F_i L_i}{A_i E_i}$$



Some more definitions

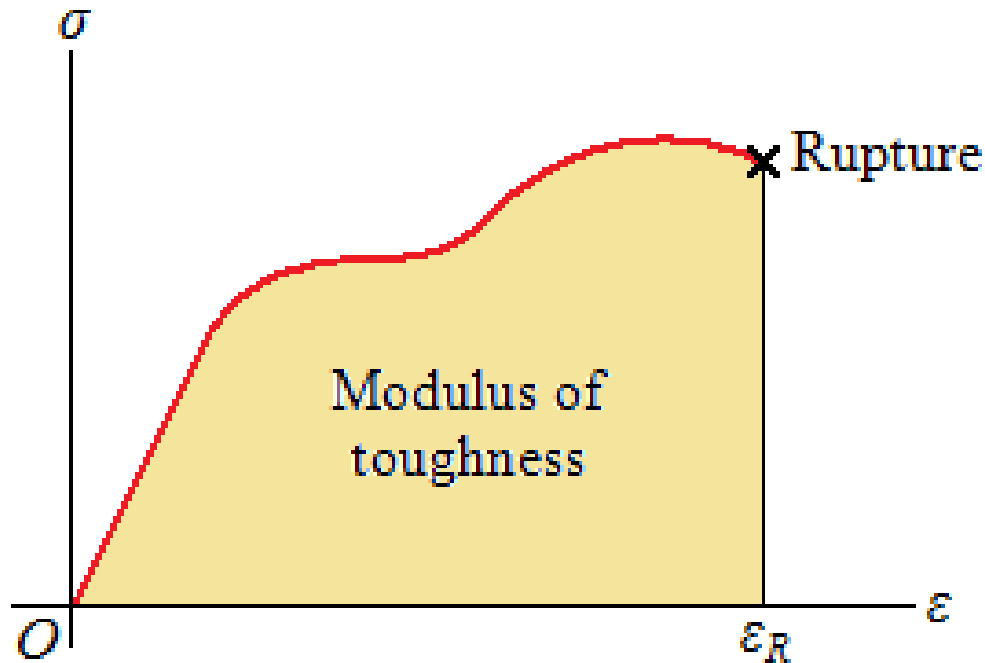
- **Modulus of resilience:** It is the energy absorbed per unit volume of the specimen upto the elastic limit.



Modulus of
resilience = $\frac{1}{2} \sigma \epsilon$

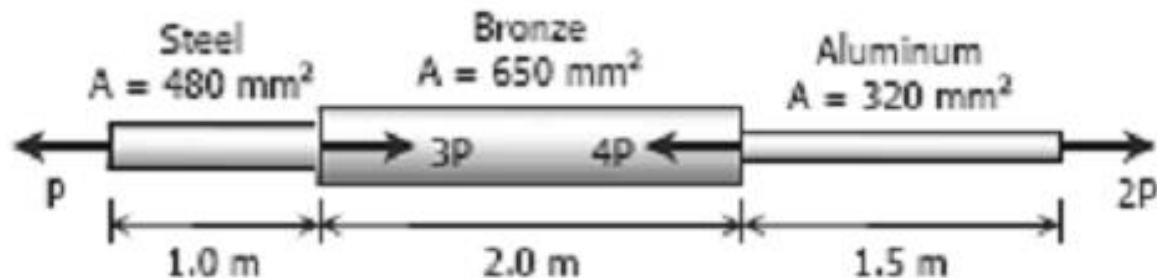
Some more definitions

- **Modulus of toughness:** It is the energy absorbed per unit volume of the specimen upto the breaking point.



Problem

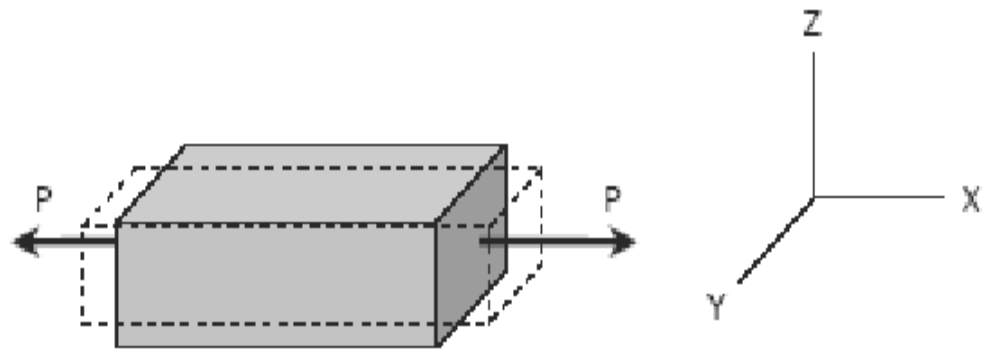
- A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $E_{br} = 83$ GPa.



Poisson's ratio

- Experiments show that if a bar is lengthened by axial tension, there is a reduction in the transverse dimensions.
- The ratio of the strains in these directions is constant for stresses within the proportional limit.
- The ratio is known as Poisson's ratio, denoted by 'ν'.

$$\nu = - \frac{\epsilon_y}{\epsilon_x} = - \frac{\epsilon_z}{\epsilon_x}$$



Poisson's ratio

- Here ϵ_x is strain in the x-direction and ϵ_y and ϵ_z are the strains in the perpendicular direction.
- The negative sign indicates a decrease in the transverse dimension when ϵ_x is positive.
- For most steel, Poisson's ratio lies in the range of 0.25 to 0.3, and 0.20 for concrete.

Poisson's ratio

- **Bi-axial deformation:** If an element is subjected simultaneously by tensile stresses, σ_x and σ_y , in the x and y directions, the strain in the x-direction due to σ_x is σ_x/E .

Simultaneously, the stress in the y direction will produce a lateral contraction on the x direction of the amount $-v\varepsilon_y$ or $-v\sigma_y/E$. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - v \frac{\sigma_y}{E} \quad \text{OR} \quad \sigma_x = \frac{(\varepsilon_x + v\varepsilon_y)E}{1 - v^2}$$

Similarly,

$$\varepsilon_y = \frac{\sigma_y}{E} - v \frac{\sigma_x}{E} \quad \text{OR} \quad \sigma_y = \frac{(\varepsilon_y + v\varepsilon_x)E}{1 - v^2}$$

Poisson's ratio

- **Tri-axial deformation:** If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x , σ_y , and σ_z , which are accompanied by strains ϵ_x , ϵ_y , and ϵ_z , respectively,

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

- Tensile stresses and elongation are taken as positive.
Compressive stresses and contraction are taken as negative.

Poisson's ratio

- The relationship between modulus of elasticity 'E', shear modulus 'G' and Poisson's ratio 'ν' is:

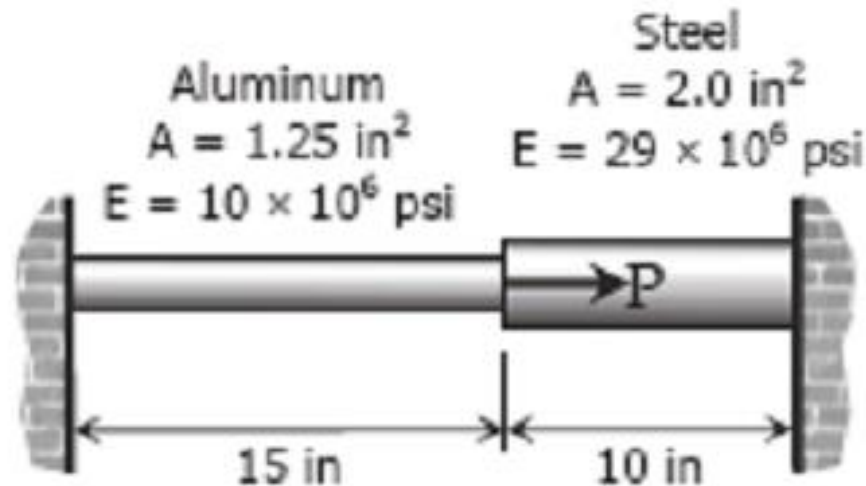
$$G = \frac{E}{2(1 + \nu)}$$

Statically indeterminate members

- There are certain conditions of axially loaded members in which the equations of static equilibrium are not sufficient for a solution.
- When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called statically indeterminate.
- These cases require the use of additional relations that depend on the elastic deformations in the members.

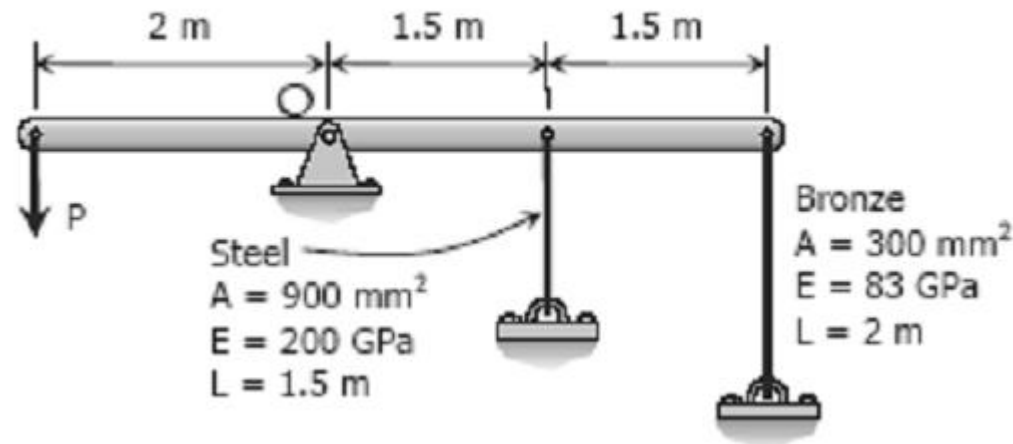
Problem

- The composite bar in Fig. is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load $P = 50$ kips.



Problem

- As shown in Fig., a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.



Thermal stress

- Temperature changes cause the body to expand or contract. The amount δ_T , is given by,

$$\delta_T = \alpha L (T_f - T_i) = \alpha L \Delta T$$

where α is the coefficient of thermal expansion in $\text{m}/\text{m}^\circ\text{C}$, L is the length in meter, and T_i and T_f are the initial and final temperatures, respectively in $^\circ\text{C}$.

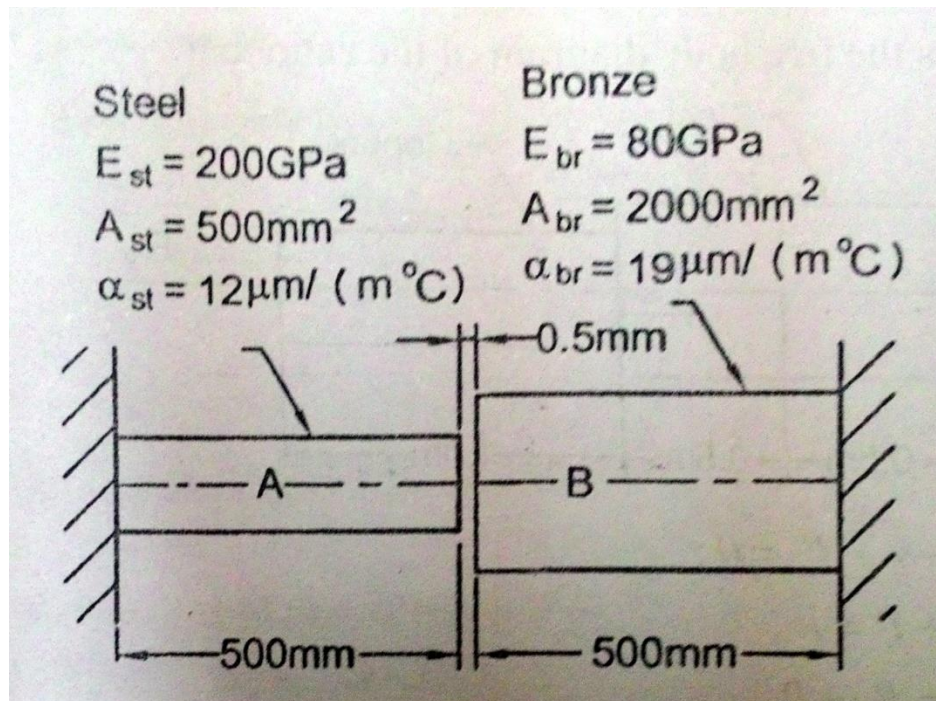
For steel, $\alpha = 11.25 \times 10^{-6} / ^\circ\text{C}$.

Thermal stress

- If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure.
- In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as **thermal stress**.

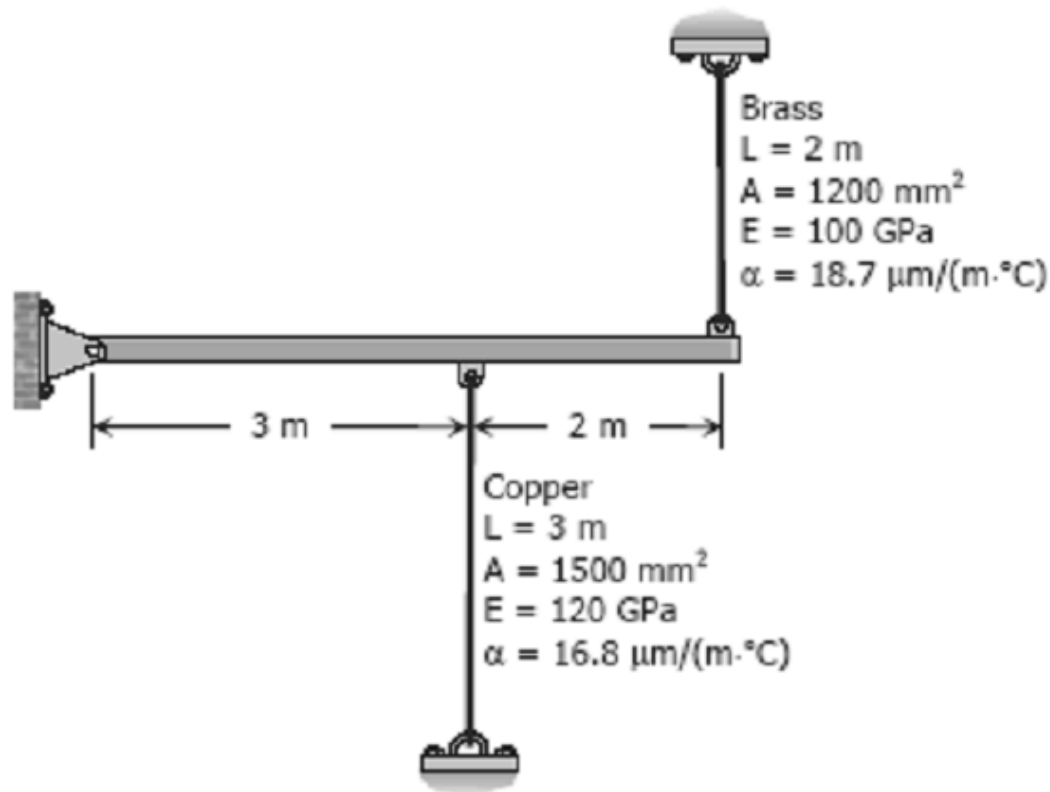
Problem

- One steel bar A and one bronze bar B are supported by the rigid walls at 0°C such that there is a gap of 0.5 mm between them as shown in fig. Determine the stress in each bar at 100°C .



Problem

- A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. If the system is initially stress-free, calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



Assignment #1

- Prob no.

247, 255, 256, 269, 271, 273 (Singer's book)